Cyclic inner functions in Bergman type spaces

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(Joint work with O. El Fallah and K. Kellay)

In this talk, we try identify, as far as possible, the cyclic vectors in the Bergman type spaces,

$$B^2_\alpha := \{f(z) = \sum_{n\geq 0} a_n z^n \text{ analytic in the unit disc } \mathbb{D} \,:\, \|f\|^2_{B^2_\alpha} = \sum_{n\geq 0} \frac{|a_n|^2}{\alpha(n)^2} < +\infty\},$$

that is, those functions f such that the polynomial multiples of f are dense in this space. Here, $(\alpha(n))_{n\geq 0}$ is a positive sequence for which $\sup_{n\geq 0} \frac{\alpha(n)}{\alpha(n+1)} < +\infty$, so that the space B^2_α is closed under multiplication by the coordinate function z. The corresponding problem for the Hardy space $H^2(\mathbb{D})$, $(\alpha \equiv 1)$, was solved in 1948 by Beurling who proved that a function f is cyclic in $H^2(\mathbb{D})$ if and only if it is an outer function. This was part of his classification of the z–invariant subspaces of $H^2(\mathbb{D})$.