Conference in honor of Professor Heydar Radjavi - Bled 05
Abstracts

Cyclic inner functions in Bergman type spaces

Abdellatif Bourhim, Laval University, Canada

(Joint work with O. El Fallah and K. Kellay)

In this talk, we try identify, as far as possible, the cyclic vectors in the Bergman type spaces,

$$B^2_\alpha := \{f(z) = \sum_{n\geq 0} a_n z^n \text{ analytic in the unit disc } \mathbb{D} \,:\, \|f\|^2_{B^2_\alpha} = \sum_{n\geq 0} \frac{|a_n|^2}{\alpha(n)^2} < +\infty\},$$

that is, those functions f such that the polynomial multiples of f are dense in this space. Here, $(\alpha(n))_{n\geq 0}$ is a positive sequence for which $\sup\limits_{n\geq 0} \frac{\alpha(n)}{\alpha(n+1)} < +\infty$, so that the space B^2_α is closed under multiplication by the coordinate function z. The corresponding problem for the Hardy space $H^2(\mathbb{D})$, ($\alpha \equiv 1$), was solved in 1948 by Beurling who proved that a function f is cyclic in $H^2(\mathbb{D})$ if and only if it is an outer function. This was part of his classification of the z–invariant subspaces of $H^2(\mathbb{D})$.

Commutativity preserving maps

Matej Brešar, University of Maribor, Slovenia

The problem of describing a linear map between algebras that maps commuting pairs of elements into commuting pairs has been an active area of research over many years in different mathematical branches (linear algebra, operator theory, ring theory). One of the leading contributors has been Heydar Radjavi.

We shall survey the development in this area and at the end present some most recent, yet unpublished results.

Completely rank-nonincreasing linear maps

Don Hadwin, University of New Hampshire, USA

I will discuss completely rank-nonincreasing linear maps. I will show how they relate to old problems and how they have been applied to obtain results in opertor algebras. I will also show how these functions lead to a new version of algebraic reflexivity. I will include a list of the main open problems in the subject.

Simultaneous Hessenberg forms

John Holbrook, University of Guelph, Canada

Some years ago Heydar Radjavi formulated an elegant criterion for matrices that are unitarily similar to a tridiagonal matrix. A key problem in this area was solved by V. Pati, using methods from algebraic geometry. We discuss the related problem of finding simultaneous Hessenberg forms for a pair of matrices (based on work with Jean-Pierre Schoch). One may ask, for example, whether there are useful criteria for simultaneous Hessenberg form in analogy with those for simultaneous triangular form. A natural question about the 4×4 case subsumes that answered by Pati.

Factoring matrices in various ways

Charles Johnson, College of William and Mary, USA

We mention recent work about factoring matrices that goes in a variety of directions: (1) existence of LU factorization and the idea of quasi-LU factorization when an LU factorization does not exist; (2) spectrally arbitrary factorization and the possibilities for Jordan structure; and (3) factorization into P-matrices and related classes.

Bi-analytic operator algebras and Euclidean families of projections

Aristides Katavolos, University of Athens, Greece

(Joint work with S.C. Power (Lancaster))

We study the families $\mathcal{L}_p,\ \mathcal{L}_h$ of projections on $L^2(\mathbb{R})$ whose ranges are simultaneously invariant under multiplication by $H^\infty(\mathbb{R})$ and also under forward translations and dilations respectively. Both these families turn out to be compact connected locally Euclidean manifolds (in the strong operator topology).

The operator algebras leaving these families invariant are both generated by non-commuting pairs of copies of $H^\infty(\mathbb{R})$. They contain no nontrivial selfadjoint or finite rank operators, but they have approximate identities consisting of Hilbert-Schmidt operators.

Their isometric automorphism groups turn out to be Lie groups, while the algebras themselves are generated by (unitary representations of) Lie semigroups.

These results suggest new directions in nonselfadjoint operator algebra theory: on the one hand to shift attention to the topological properties of invariant projection families and on the other to study algebras generated by Lie semigroups.

Nearly commutative matrices

Thomas J. Laffey, University College Dublin, Ireland

We consider pairs of complex $n \times n$ matices (A, B).

For $n > 1$, clearly if A and B commute, then they cannot generate the full matrix algebra, $M_n(\mathbb{C})$. We present a number of results showing that, under several interpretations of the term "nearly commuting", we can find nearly commuting pairs (A, B) which do generate the full matrix algebra. Many of the results are based on the representation theory of finite groups,particularly that of finite p-groups and the symmetric group and finite dimensional division algebras.

We also present a number of open questions.

Playing with the bands in the 60's - Heydar Radjavi and semigroups of idempotents

Leo Livshits, Colby College, USA

We will give a brief survey of Radjavi's own and Radjavi-inspired contributions to the theory of semigroups of idempotents.

When are two C*-algebras Jordan isomorphic?

Martin Mathieu, Queen's University Belfast, Northern Ireland

From its very beginnings the theory of C^*-algebras has been closed related to Quantum Mechanics, since the observables in a quantum mechanical system can be described by self-adjoint operators on Hilbert space. However, as was observed by Jordan, von Neumann and Wigner, the Jordan algebra structure of a C^*-algebra is even more closely tied to the physical situation. Thus, from the point of view of Physics at least, a natural question is when two C^*-algebras are isomorphic as Jordan algebras. (Under special circumstances this may then even imply that they are isomorphic as C^*-algebras.)

One the best known and most important results is Kadison's theorem stating that a unital surjective isometry between C^*-algebras must be a Jordan $*$-algebra isomorphism. We study the question whether there is an analogous chracterisation of non-selfadjoint Jordan algebra isomorphisms. Maybe not surprising this leads us to spectral theory in C^*-algebras and mappings preserving the spectral radius.

Rank and reflexivity of operator spaces

Roy Meshulam, Technion - Israel Institute of Technology, Israel

(Joint work with P. Šemrl)

A well-known result of Larson asserts that a finite-dimensional operator space that contains no finite rank operators, is reflexive. Quantitative versions were obtained by Ding and Li and Pan. We'll describe an improvement of these results, together with some examples and conjectures concerning its optimality.

Invariant subspaces for polynomially bounded operators

Vladimir Müller, Czech Academy of Sciences, Czech Republic

(Joint work with C. Ambrozie)

By a well-known result by Brown, Chevreau and Pearcy, every Hilbert space contraction whose spectrum contains the unit circle has a non-trivial invariant subspace. A generalization of this result to the Banach space setting will be presented. If T is a polynomially bounded operator on a Banach space X whose spectrum contains the unit circle then its adjoint T^* has a nontrivial invariant subspace. In particular, if the space X is reflexive then T has an invariant subspace.

C. Ambrozie, V. Müller: Invariant subspaces for polynomially bounded operators, J. Funct. Anal. 213 (2004), 321–345.

Extreme points of stochastic operators

Eric Nordgren, University of New Hampshire, USA

The idea of a stochastic matrix can be carried over to operators on certain L^2 spaces. The familiar characterization of the extreme points of the stochastic matrices is seen to hold in the L^2 setting as well. We also consider other versions of stochastic operators.

Operators with numerical range in a halfplane

Leiba Rodman, College of William and Mary, USA

(Joint work with W. S. Cheung and C. K. Li)

Various characterizations are given for real and complex Hilbert space operators whose numerical range is contained in a halfspace. In particular, one characterization is given in terms of low dimensional pieces of the operators involved. As it turns out, the real case is not entirely analogous to the complex case.

A survey of Heydar Radjavi

Peter Rosenthal, University of Toronto, Canada

A brief look at the life and work of a fine mathematician and a wonderful human being.

Around the Fuglede theorem

Victor Shulman, Vologda Technical University, Russia

The famous Fuglede Theorem states that linear operator equations $AX + XB = 0$ and $A^*X + XB^* = 0$ are equivalent on the space of all bounded operators between Hilbert spaces, if A and B are bounded normal operators. There will be discussed various results on linear operator equations, multiplication operators, ideals in Banach algebras, spectral synthesis that are in some way related to this theorem.

The semigroup generated by a unitary orbit

Ahmed Sourour, University of Victoria, Canada

(Joint work with Heydar Radjavi)

For a singular matrix A we determine the mutilpicative semigroup generated by the unitary orbit of A.

Adjacency preserving maps on hermitian matrices

Peter Šemrl, University of Ljubljana, Slovenia

Two hermitian matrices A and B are said to be adjacent if $\text{rank}\,(A-B) = 1$. Maps on hermitian matrices preserving adjacency are important because of applications in physics, complex analysis, and the theory of linear preservers. Under some mild conditions such maps were characterized by Hua. A short proof of Hua's theorem was given by Radjavi and Šemrl. We will present a recently obtained improvement of this result.

Between Newton and Hilbert

Jaroslav Zemánek, Polish Academy of Sciences, Poland

(Joint work with Alfonso Montes-Rodríguez and Juan Sánchez-Álvarez)

A characterization of the Hilbert space of (square) integrable functions in terms of their primitives (originating from Newton) will be presented.