

Geometry and means

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Positive definite matrices play a significant role in various mathematical settings: as covariance matrices in statistics, as transfer matrices in systems engineering, as factors in the polar decomposition of tensors used in continuum physics, and as density matrices in quantum mechanics, for example. It is often necessary to consider the average or "mean" of several of such matrices. In general the matrices do not commute and this makes it difficult to define a good notion of geometric mean (GM). In fact, there are a number of competing definitions for the GM. We'll discuss a natural Riemannian geometry on the space of positive definite matrices and consider variants of the GM that are based on that non-Euclidian geometry.