

Mathematics of medical imaging

Ali Jafarian, University of New Haven, USA

One of the main topics in Medical Imaging is Computerized Tomography (CT), which involves reconstruction of cross-sectional images of an object, and consequently its whole image. This is done by using attenuation coefficient (function), which is a function that quantifies the tendency of the object to scatter or absorb an x-ray of a given energy. The attenuation function is unknown, but its integrals along different line segments in the cross section can be determined using Beers Law. This requires a great deal of computation of line or plane integrals. From a pure mathematical point of view, the problem is reconstruction of a function, defined on a compact region in a plane, from the knowledge of its integrals along many different line segments in the region. In 1917, J. Radon solved this mathematical problem in a different context. No real application of the Radon work (Radon Transform) was known until early 1970s. At that time, G.N. Hounsfield used Radon Transform to invent an x-ray computerized tomographic scanner, for which he received a Nobel Prize in 1972. (Hounsfield shared the prize with Allan Cormack, who independently discovered some of the algorithms.)

A lot of mathematics, including Geometry, Linear Algebra, Operator Theory, Fourier and Wavelet Transforms, is involved in Medical Imaging. There has been a lot of innovations not only in the design of the machines, but also in the algorithms and techniques used to recover, filter noise, and improve the quality and clarity of the pictures, as well as making the process faster. In early 1990s, wavelets found their way in medical imaging. They are used, instead of the Fourier Transform, in the inverse problem; i.e., the reconstruction of the image from the data captured by the detectors. Also, wavelets have been used for improving the quality of the image and the speed.

* This is an expository talk and is based on a recent study that I had on the subject.