

## Clarkson-McCarthy inequalities in Schatten ideals

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For Schatten ideals  $C_p$  of compact operators on a Hilbert space  $H$ , denote by  $C_p(n)$  the spaces of all columns  $\overline{A} = (A_i)$ ,  $1 \leq i \leq n < \infty$ , with  $A_i \in C_p$ . Let  $H^n$  be the sum of  $n$  copies of  $H$ . Each  $R \in B(H^n)$  can be represented as an  $n \times n$  block-matrix operator  $(R_{jk})$  with  $R_{jk} \in B(H)$ , so it acts on  $C_p(n)$ . We prove several inequalities which link the  $\|\cdot\|_p$ -norms of operators from  $R\overline{A}$  and from  $\overline{A}$ . For operators  $R = (r_{jk}\mathbf{1})$ , where  $r_{jk}$  are some  $n$ th roots of unity and  $\mathbf{1}$  is the identity operator on  $H$ , some of these inequalities were previously established by Bhatia and Kittaneh. For  $n = 2$ , Bhatia and Kittaneh's inequalities are, in turn, generalizations of the Clarkson-McCarthy and Hirzallah-Kittaneh inequalities for sums of operators.

Additionally, let  $\{P_j\}_{j=1}^n$  be a set of mutually orthogonal projections in  $B(H)$  with  $\sum_{j=1}^n P_j = \mathbf{1}$ , and let  $\{Q_k\}_{k=1}^m$  be another such set. We then obtain some inequalities which link the norm  $\|A\|_p$ , for  $A \in C_p$ , and the norms  $\|P_j A Q_k\|_p$ . This extends the norm inequalities for partitioned operators hitherto proven by Bhatia and Kittaneh.