Linear Algebra Workshop - Bled 05
Abstracts

Applications of Kazhdan's property to automatic continuity

Jeronimo Alaminos, University of Granada, Spain

Let G be a compact group which has the strong Kazhdan's property (T) and let τ be an isometric representation of G on $L^p(\mu)$ for some measure space (Ω, Σ, μ) and $1 < p < \infty$. We characterize the continuity of all the linear operators from $L^p(\mu)$ which commute with translations and the property of $\|\cdot\|_p$ being the unique norm (up to equivalence) on $L^p(\mu)$ with respect to which all translations are continuous in terms of the finite-dimensionality of the subspace of all invariant functions.

Joint invariant subspaces for tuples of polynomially bounded operators

Calin Ambrozie, Czech Academy of Sciences, Czech Republic

(Joint work with Vladimir Müller)

We present some results of existence of the joint invariant subspaces for commuting n-tuples $T = (T_1, \ldots, T_n)$ of bounded linear operators $T_j : X \to X$ on a complex Hilbert or Banach space X. The main result states, for $X =$ a Hilbert space, that if the Taylor joint spectrum of T is dominant in the unit polydisc $D = \{z \in C^n : |z_j| < 1\}$ and T is polynomially contractive (namely $\|p(T)\| \leq \sup_D |p|$ for all polynomials $p \in C[z]$) and of class C_{00}, then T has jointly invariant closed linear subspaces. The similar result holds as well for T polynomially bounded on a Banach space X, provided that the essential Taylor joint spectrum of T be dominant in D.

Applications of Fourier analysis in matrix theory

Rajendra Bhatia, Indian Statistical Institute, India

Fourier analysis has been applied in interestig matrix problems in several ways. Some of these are calculation of norms, solution of matrix equations, perturbation of eigenspaces, positive definiteness, matrix inequalities.

We will illustrate such applications by a few examples from this list.

Some eigenvalue problems for the p-Laplacian

Paul Binding, University of Calgary, Canada

The p-Laplacian eigenvalue equation is nonlinear, but there are enough remains of linearity to make it interesting for linear people. The talk will cover some sources of the equation, review various results which do carry over (with modification) from the linear ones, and raise a few questions, for example about nonlinear algebra.

Rotation invariant norms

Jose Extremera Lizana, University of Granada, Spain

If $N \geq 2$, then there exist finitely many rotations of the sphere $\mathbb{S}^N$ such that the set of the corresponding rotation operators on $L^p(\mathbb{S}^N)$ determines the norm topology of that space for $1 < p < \infty$. For $N = 1$ the situation is different: for any N-set of $\mathbb{T}$ the set of the corresponding rotation operators on $L^2(\mathbb{S}^1)$ does not determine the norm topology of that space.

Geometry and means

John Holbrook, University of Guelph, Canada

(Joint work with Rajendra Bhatia, Indian Statistical Institute, India)

Positive definite matrices play a significant role in various mathematical settings: as covariance matrices in statistics, as transfer matrices in systems engineering, as factors in the polar decomposition of tensors used in continuum physics, and as density matrices in quantum mechanics, for example. It is often necessary to consider the average or "mean" of several of such matrices. In general the matrices do not commute and this makes it difficult to define a good notion of geometric mean (GM). In fact, there are a number of competing definitions for the GM. We'll discuss a natural Riemannian geometry on the space of positive definite matrices and consider variants of the GM that are based on that non-Euclidian geometry.

On a class of projections on $*$-rings

Dijana Ilišević, University of Zagreb, Croatia

(Joint work with Maja Fošner, University of Maribor, Slovenia)

This work is motivated by the study of bicircular projections on some matrix and operator spaces, carried out by Stachó and Zalar. A projection on a complex Banach space is called bicircular if its linear combinations with the complementary projection having coefficients of modulus one are isometries. In this talk we first describe the structure of projections $P : R \to R$, where R is a 2-torsion free semiprime $*$-ring, satisfying the functional identity

$$P(xyx) = P(x)yx - xP(y^*)^*x + xyP(x)$$

for all $x, y \in R$. Then we apply the obtained results to bicircular projections on C*-algebras.

Mathematics of medical imaging

Ali Jafarian, University of New Haven, USA

One of the main topics in Medical Imaging is Computerized Tomography (CT), which involves reconstruction of cross-sectional images of an object, and consequently its whole image. This is done by using attenuation coefficient (function), which is a function that quantifies the tendency of the object to scatter or absorb an x-ray of a given energy. The attenuation function is unknown, but its integrals along different line segments in the cross section can be determined using Beers Law. This requires a great deal of computation of line or plane integrals. From a pure mathematical point of view, the problem is reconstruction of a function, defined on a compact region in a plane, from the knowledge of its integrals along many different line segments in the region. In 1917, J. Radon solved this mathematical problem in a different context. No real application of the Radon work (Radon Transform) was known until early 1970s. At that time, G.N. Hounsfield used Radon Transform to invent an x-ray computerized tomographic scanner, for which he received a Nobel Prize in 1972. (Hounsfield shared the prize with Allan Cormack, who independently discovered some of the algorithms.)

A lot of mathematics, including Geometry, Linear Algebra, Operator Theory, Fourier and Wavelet Transforms, is involved in Medical Imaging. There has been a lot of innovations not only in the design of the machines, but also in the algorithms and techniques used to recover, filter noise, and improve the quality and clarity of the pictures, as well as making the process faster. In early 1990s, wavelets found their way in medical imaging. They are used, instead of the Fourier Transform, in the inverse problem; i.e., the reconstruction of the image from the data captured by the detectors. Also, wavelets have been used for improving the quality of the image and the speed.

* This is an expository talk and is based on a recent study that I had on the subject.

Clarkson-McCarthy inequalities in Schatten classes

Edward Kissin, London Metropolitan University, Great Britain

For Schatten ideals C_p of compact operators on a Hilbert space H, denote by $C_p(n)$ the spaces of all columns $\overline{A} = (A_i)$, $1 \leq i \leq n < \infty$, with $A_i \in C_p$. Let H^n be the sum of n copies of H. Each $R \in B(H^n)$ can be represented as an $n \times n$ block-matrix operator (R_{jk}) with $R_{jk} \in B(H)$, so it acts on $C_p(n)$. We prove several inequalities which link the $\| \cdot \|_p$-norms of operators from $R\overline{A}$ and from $\overline{A}$. For operators $R = (r_{jk}\mathbf{1})$, where r_{jk} are some nth roots of unity and $\mathbf{1}$ is the identity operator on H, some of these inequalities were previously established by Bhatia and Kittaneh. For $n = 2$, Bhatia and Kittaneh's inequalities are, in turn, generalizations of the Clarkson-McCarthy and Hirzallah-Kittaneh inequalities for sums of operators.

Additionally, let $\{P_j\}_{j=1}^n$ be a set of mutually orthogonal projections in $B(H)$ with $\sum_{j=1}^n P_j = \mathbf{1}$, and let $\{Q_k\}_{k=1}^m$ be another such set. We then obtain some inequalities which link the norm $\|A\|_p$, for $A \in C_p$, and the norms $\|P_jAQ_k\|_p$. This extends the norm inequalities for partitioned operators hitherto proven by Bhatia and Kittaneh.

Symmetic nonnegative matrices

Thomas J. Laffey, University College Dublin, Ireland

We say that a list σ of n complex numbers is *realizable* if it is the spectrum of an entry-wise nonnegative $n \times n$ matrix A, and we say that σ is *symmetrically realizable* if A can be chosen to be symmetric.

Many results are available in the literature describing ways of perturbing and combining realizable spectra while preserving realizability. We consider the corresponding problems for symmetric realizability, and, in particular, we will discuss the symmetric analogues of results of Brauer, Fiedler, Smigoc and Guo Wuwen.

We will also discuss the current status of the general problem of characterizing nonnegative symmetric matrices via their spectra.

Automorphisms of M_n, partially ordered by the Drazin star order

Peter Legiša, University of Ljubljana, Slovenia

Let M_n be the space of all $n \times n$ matrices with coefficients in $\mathbb{R}$ or $\mathbb{C}$, where $n \geq 3$. The Drazin star order on M_n is defined by

$$A \leq^* B \quad \text{iff} \quad \mathrm{A^*A = A^*B} \quad \text{and} \quad \mathrm{AA^* = BA^*},$$

where A^* is the Hermitian adjoint (i.e. the conjugate transpose) of A. We characterize surjective mappings Φ on M_n such that

$$A \leq^* B \quad \text{iff} \quad \Phi(\mathrm{A}) \leq^* \Phi(\mathrm{B}) \qquad (\mathrm{A, B \in M_n}).$$

The tools we use are the Fundamental theorem of projective geometry, Uhlhorn's theorem, and the Penrose decomposition, which we need to describe the main result as well.

Skew lattices of idempotents in rings, especially in matrix rings

Jonathan Leech, Westmont College, USA

Skew lattices can arise in rings as multiplicative bands of idempotents that are especially well-behaved. The talk will consider both properties of such skew lattices as well as various classes of examples. Particular attention will be given to examples in matrix rings and to results about them.

Distance preserving maps on matrices

Chi-Kwong Li, College of William and Mary, USA

Let $\mathcal{M}$ be a linear space of matrices equipped with a multiplication $A * B$. Suppose $\|\cdot\|$ is a norm on $\mathcal{M}$ satisfying certain invariant property such as unitarily invariant, unitary similarity invariant, or unitary congruence invariant. We discuss results and open problems concerning mappings $T : \mathcal{M} \to \mathcal{M}$ which satisfy one of the following conditions.

1. T is linear and $\|T(A)\| = \|A\|$ for all $A \in \mathcal{M}$.

2. $\|T(A) - T(B)\| = \|A - B\|$ for all $A, B \in \mathcal{M}$.

3. $T(A * B) = T(A) * T(B)$ for all $A, B \in \mathcal{M}$, and $\|T(A)\| = \|A\|$ for all $A \in \mathcal{M}$.

4. $\|T(A) * T(B)\| = \|A * B\|$ for all $A, B \in \mathcal{M}$.

Furthermore, we consider similar problems for mappings between different matrix spaces $\mathcal{M}_1$ and $\mathcal{M}_2$ that are equipped with different norms.

Complete decomposability of nearly commuting semigroups of non-negative matrices

Leo Livshits, Colby College, USA

(Joint work with G. MacDonald and H. Radjavi)

A matrix semigroup is completely decomposable if it is simultaneously similar to a semigroup of upper triangular matrices via a permutation similarity. We present several criteria for complete decomposability of semigroups of entry-wise non-negative matrices. For example, if such a semigroup is almost abelian then it is completely decomposable exactly when each matrix in the semigroup is completely decomposable. Similarly, if a collection of entry-wise non-negative matrices is close to a completely decomposable collection, it is itself completely decomposable. If time permits we will show how these results are extended to non-negative compact operators on $\mathcal{L}^p(X, \mu)$.

The exponent of a K-primitive matrix

Raphael Loewy, Technion - Israel Institute of Technology, Israel

Let K be a proper, convex cone in $\mathbb{R}^n$. Let $\pi(K)$ denote the set of all K-nonnegative matrices, that is, all $A \in \mathbb{R}^{n\times n}$ which map K into K. We say that $A \in \pi(K)$ is *K-positive* provided that $Ax \in intK$ for every $0 \neq x \in K$, and it is *K-primitive* provided that there exists a positive integer ℓ such that A^ℓ is K-positive. The *exponent* of a K-primitive matrix is the smallest positive integer ℓ such that A^ℓ is K-positive.

When $K = \mathbb{R}^n_+$, the nonnegative orthant, the notions of a K-nonnegative, K-positive and K-primitive matrix reduce to the well known notions of a nonnegative (elementwise), positive and primitive matrix.

It is our purpose to discuss K-primitive matrices, and in particular their exponents. An important tool in the discussion is a directed graph that can be associated with each $A \in \pi(K)$, and whose vertices are the extreme rays of K. When K is polyhedral and A is K-primitive, an upper bound for the exponent of A is obtained. This bound generalizes the well known result, due to Wielandt, which asserts that the exponent of an $n \times n$ primitive matrix is at most $n^2 - 2n + 2$.

Around Burnside's theorem on operator algebras

Victor Lomonosov, Kent State University, USA

Burnside's very well known and very useful theorem states that the only irreducible algebra of linear transformations on a vector space of finite dimension greater than 1 over an algebraically closed field is the algebra of all linear transformations on the space. We give a simple proof of the Burnside's Theorem.

More or less local multipliers

Martin Mathieu, Queen's University Belfast, Northern Ireland

In this talk we report on work in progress with Pere Ara, Barcelona. Building on the theory of local multipliers as developed in our monograph in Springer-Verlag (2003) we introdcue the notion of a maximal C^*-algebra of quotients. We study the relation with the local multiplier algebra and with the injective envelope of a C^*-algebra (a la Haman-Paulsen) and present a few first results.

Laplacians, homology and hypergraph matching

Roy Meshulam, Technion - Israel Institute of Technology, Israel

(Joint work with R. Aharoni and E. Berger)

The spectral gap of a graph is an important parameter that quantifies various connectivity properties of the graph. We'll discuss spectral gaps of higher dimensional complexes, with some applications in topological combinatorics. One consequence is a lower bound on the homological connectivity of the independence complex of a graph. This in turn implies Hall type theorems for matchings in hypergraphs.

Growth and identities of semigroups of matrices

Jan Okniński, University of Warsaw, Poland

Finitely generated subsemigroups $S \subseteq M_n(K)$ of the multiplicative monoid of $n \times n$ matrices over a field K are considered. If $S = \langle a_1, \ldots, a_n \rangle$, then the corresponding growth function is defined by $d(n) =$ the number of elements of S that can be presented as words of length at most n in the generators $a_1, \ldots, a_n$. Classical results on the growth of groups and partial results on the growth of semigroups of matrices will be presented. Connections between the growth rate of S, the identities satisfied in S and the structure of S will be discussed.

Matrix-theoretic methods in the analytic theory of polynomials

Rajesh Pereira, University of Saskatchewan, Canada

We show how classical matrix theoretical techniques can be used to study the relationship between the zeros of a polynomial and those of its derivative.

On collections of matrices with entries from a division ring

Mehdi Radjabalipour, University of Kerman, Iran

The paper surveys recent results by H. Radjavi, P. Rosenthal, B.R. Yahaghi, H. Momenaee-Kermani, and the author about extensions of various results on transitivity, irreducibility, and triangularizability of collections of matrices with entries from a division ring D. The division ring may range from the most general case to the case that $[D; F]$ is finite or the most special case that D is the field of real quaternions. Also, the matrix collection may be a semigroup, an algebra over the center of D, or a left D-module ring.

Semitransitive collections of operators

Heydar Radjavi, University of Waterloo, Canada

A collection F of operators on a vector space V is called semitransitive if given x and y in V there is a member A of F such that either $Ax = y$ or $Ay = x$. For a group, or more generally, any set of invertible operators that contains inverses of all its members, this concept clearly coincides with transitivity. (An obvious topological version of the definition can also be made when V is a normed space.) In the last few years, work has been done on semitransitive algebras and semigroups of operators, including the joint work first by H. Rosenthal and V. Troitsky, and then by J. Bernik, L. Grunenfelder, M. Mastnak, V. Troitsky, and the present author. More recently, I have also worked with Troitsky on semitransitive spaces of operators. I will report on some of these results.

Noncanonical factorization

Leiba Rodman, College of William and Mary, USA

(Joint work with C. V. M. van der Mee, I. M. Spitkovsky, and H. J. Woerdeman)

Noncanonical factorizations are studied in the abstract context of matrix valued functions defined on connected compact abelian groups, with respect to a total order on the dual abelian group. The basic examples are the dual group of integers and the dual group of reals with the discrete topology (with the natural total orders). Criteria for factorizability for some classes of matrix functions are given. Several open problems will be stated. Applications include systems of difference equations with delays, orthogonal polynomials, and Toeplitz operators.

On polynomial numerical hulls of matrices and matrix polynomials

Abbas Salemi, University of Kerman, Iran

The notion of polynomial numerical hull was introduced by O. Nevanlinna in 1993. In this note we study the polynomial numerical hulls of normal matrices and the relationship between rectangular hyperbolas and polynomial numerical hulls of order two for normal matrices. Also we introduce the polynomial numerical hulls of matrix polynomials.

Spectra of multiplication operators on Banach algebras

Victor Shulman, Vologda Technical University, Russia

Let A be a Banach algebra and E - the algebra of all multiplication operators (elementary operators) on A. We look for the conditions under which the inclusions $\sigma(T+S) \subset \sigma(T)+\sigma(S)$ for all $T, S \in E$. The results we obtain have applications to the problem of the existence and "elementwise" characterization of the maximal Engel ideal and maximal E-solvable ideal in a Lie algebra of compact operators (the natural analogues of the nil-radical and radical of a finite-dimensional Lie algebra).

Products of idempotents

Ahmed Sourour, University of Victoria, Canada

(Joint work with W. Longstaff)

We determine the class of operators on an infinite-dimensional Hilbert space that are products of idempotents. In particular every operator that has infinite dimensional kernel annd co-kernel is a product of 3 idempotents.

Some formulae for norms of elementary operators

Richard Timoney, Trinity College Dublin, Ireland

We present a formula for the norm of an elementary operator on a C^*-algebra that seems to be new. The formula involves (matrix) numerical ranges and a kind of geometrical mean for positive matrices, the tracial geometric mean, which seems not to have been studied previously and has interesting properties. The talk will focus on the properties of the tracial geometric mean more than on the elementary operators.

Topological radicals of normed algebras

Yuri Turovskii, Academy of Sciences of Azerbaijan, Azerbaijan

(Joint work with Victor Shulman)

We consider the axiomatics of topological radicals, some procedures over topological radicals, the most important examples and present the results on tensor radicality and joint spectral radius formulae.

Banach algebras generated by compact groups of operators

Armando R. Villena, University of Granada, Spain

We prove that if τ is a strongly continuous representation of a compact group G on a Banach space X, then the weakly closed Banach algebra generated by the Fourier transforms $\int_G \tau(t)d\mu(t)$ with $\mu \in M(G)$ is a semisimple Banach algebra.

Powers of operators and the numerical range

Jaroslav Zemánek, Polish Academy of Sciences, Poland

(Joint work with Iwona Wróbel)

It is shown that a Hilbert space operator A is power-bounded if and only if the sums $A^k + A^{*k}$ are bounded for $k = 1, 2, \ldots$. The method is based on the concept of numerical range, the Schur triangularization of matrices, and a recent inequality of Kittaneh. Its analytic development yields analogous results for arbitrary families of operators, in particular, for the Cesàro means of powers. Related open problems arise throughout.

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Student presentations

Jordan homomorphisms on triangular matrices

Dominik Benkovič, University of Maribor, Slovenia

Let $\mathcal{T}_n(\mathcal{C})$ be the algebra of all $n \times n$ upper triangular matrices over a commutative unital ring $\mathcal{C}$. We describe the structure of Jordan homomorphisms from $\mathcal{T}_n(\mathcal{C})$ into an arbitrary algebra over $\mathcal{C}$. As an application we show that every Jordan derivation from $\mathcal{T}_n(\mathcal{C})$ into an $\mathcal{T}_n(\mathcal{C})$-bimodule is the sum of a derivation and an antiderivation.

Permutation-like groups of matrices

Grega Cigler, University of Ljubljana, Slovenia

We introduce the permutation-like groups, i.e., the finite groups $\mathcal{G}$ of complex matrices, such that every matrix $X \in \mathcal{G}$ is similar to a permutation matrix. We investigate the following question: When a permutation-like group is equivalent (simultaneously similar) to a group of permutation matrices? Various examples show that in general a permutation-like group does not have to be equivalent to a group of permutation matrices. In fact there are counterexamples for every $n \geq 6$. We give complete answers for low-dimensional cases $n = 2, 3, 4, 5$.

Cubic skew lattices in rings of matrices

Karin Cvetko-Vah, University of Ljubljana, Slovenia

A multiplicative band in a ring of matrices is called a ∇-band if it is closed under the cubic join operation $x \nabla y = x + y + xy - xyx - yxy$. If operation ∇ is associative on a given ∇-band S, then S forms a *cubic skew lattice*. We will give an example of a ∇-band in $M_n(F)$ that does not yield a skew lattice, state an exact condition for a pure ∇- band to form a skew lattice, and examine certain properties of cubic skew lattices in rings (of matrices).

Order preserving maps on the poset of upper triangular idempotent matrices

Ajda Fošner, University of Maribor, Slovenia

We obtain the general form of bijective order and orthogonality preserving maps on the poset of all $n \times n$ upper triangular idempotent matrices over an arbitrary field $\mathbb{F}$. We also show that the assumption of surjectivity can be removed if every nonzero homomorphism $g : \mathbb{F} \to \mathbb{F}$ is surjective.

Homomorphisms of matrix semigroups

Damjana Kokol Bukovšek, University of Ljubljana, Slovenia

We study non-degenerate homomorphisms from the multiplicative semigroup of all n-by-n matrices over a field to the semigroup of m-by-m matrices over the same field. In the case $n = 2$ and $m = 3$ we characterise all such homomorphisms. Then we restrict to an algebraically closed field of characteristic zero. We characterise all non-degenerate irreducible homomorphisms in the case $n = 2$ and $m = 4$. If $n = 2$ and a non-degenerate irreducible homomorphism maps a cyclic unipotent to a cyclic unipotent it is the composition of a symmetric power, a field homomorphism used entrywise, and a matrix conjugation. If $n \geq 3$ and $m = n + 1$, every non-degenerate homomorphism is reducible.

On covariant maps of matrices

Tatiana Shulman, Journal of Functional Analysis and its Applications, Staff assistant manager, Moscow, Russia

The classes of maps in the space of hermitian matrices that commute with the action of the unitary group by conjugation are considered. Such maps we call covariant maps.

The most popular examples of covariant maps come from the functional calculus. Namely if ϕ is a Borel function then the map $T \to \phi(T)$ is clearly covariant. We describe all covariant maps and explore their analytic properties (in particular, we find conditions under which these maps are continuous, Lipschitz or differentiable).

We consider also the maps in the matrix space that commute with the action of the general linear group by conjugation. Some applications to functional calculus are obtained.

Necessary conditions for the existence of a common quadratic Lyapunov function

Helena Šmigoc, Hamilton Institute, Maynooth, Ireland

(Joint work with Thomas J. Laffey)

The problem of determining conditions for the existence of a common quadratic Lyapunov function (CQLF) for stable switched systems is important in applied and theoretical research. We will introduce the problem and present necessary conditions for the existence of a CQLF. Those conditions are also sufficient in some special cases. We will show that the conditions are not sufficient in general.

Orbits of operators tending to infinity

Jan Vršovský, Charles University in Prague, Czech Republic

(Joint work with Vladimír Müller)

Let T be a bounded linear operator. We examine a problem whether there is a unit vector x such that $\|T^n x\| \to \infty$. Using Keith Ball's plank theorems, we prove that on a Banach space, a sufficient condition is $\sum_{n=1}^{\infty} \frac{1}{\|T^n\|} < \infty$, while on a complex Hilbert space, it is $\sum_{n=1}^{\infty} \frac{1}{\|T^n\|^2} < \infty$. The above results are the best possible. We also show analogous results for weak orbits.

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Themes for Workshop

Heydar Radjavi, University of Waterloo, Canada

- Approximate Spectral Conditions.

  Recently, there have been studies on weakened and "approximate" forms of certain spectral conditions previously known to imply commutativity, reducibility, or simultaneous triangularizability of semigroups of operators on a complex vector space. For example, Janez Bernik and I have proved that for compact groups of operators the condition

  $r(AB - BA) < 3^{1/2}$ for all A and B in the group

  (where r denotes the spectral radius) implies commutativity of the group; the previous stronger condition, used by Guralnick and giving the same conclusion was $r(AB - BA) = 0$ for all A and B. For a large class of groups, a suitably normalized condition yields reducibility of the group. Another instance of this type of study is a recent work by L. Marcoux, M. Mastnak, and the writer of these lines, generalizing the well known theorem of Specht. The latter says that if every word in A and A^* has the same trace as that word in B and B^*, then A and B are unitarily equivalent. We considered the situation where these traces are merely assumed to be "close." The simplest case is that of unitary A and B for which it is sufficient to assume that the difference between the said traces be at most 1. There are many spectral conditions whose approximate forms could be studied for various collections of operators.

- Transitivity and Related Questions.

  As my talk will indicate, there are questions on transitivity and semitransitivity of various collections, e.g. semigroups or linear spaces of operators on a finite-dimensional space, that arise naturally. Some of these have interesting "k-step" forms that can be studied.