Distance preserving maps on matrices

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Let $\mathcal{M}$ be a linear space of matrices equipped with a multiplication $A * B$. Suppose $\|\cdot\|$ is a norm on $\mathcal{M}$ satisfying certain invariant property such as unitarily invariant, unitary similarity invariant, or unitary congruence invariant. We discuss results and open problems concerning mappings $T : \mathcal{M} \to \mathcal{M}$ which satisfy one of the following conditions.

1. T is linear and $\|T(A)\| = \|A\|$ for all $A \in \mathcal{M}$.
2. $\|T(A) - T(B)\| = \|A - B\|$ for all $A, B \in \mathcal{M}$.
3. $T(A * B) = T(A) * T(B)$ for all $A, B \in \mathcal{M}$, and $\|T(A)\| = \|A\|$ for all $A \in \mathcal{M}$.
4. $\|T(A) * T(B)\| = \|A * B\|$ for all $A, B \in \mathcal{M}$.

Furthermore, we consider similar problems for mappings between different matrix spaces $\mathcal{M}_1$ and $\mathcal{M}_2$ that are equipped with different norms.