The exponent of a K-primitive matrix

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Let K be a proper, convex cone in $\mathbb{R}^n$. Let $\pi(K)$ denote the set of all K-nonnegative matrices, that is, all $A \in \mathbb{R}^{n\times n}$ which map K into K. We say that $A \in \pi(K)$ is K-*positive* provided that $Ax \in intK$ for every $0 \neq x \in K$, and it is K-*primitive* provided that there exists a positive integer ℓ such that A^ℓ is K-positive. The *exponent* of a K-primitive matrix is the smallest positive integer ℓ such that A^ℓ is K-positive.

When $K = \mathbb{R}^n_+$, the nonnegative orthant, the notions of a K-nonnegative, K-positive and K-primitive matrix reduce to the well known notions of a nonnegative (elementwise), positive and primitive matrix.

It is our purpose to discuss K-primitive matrices, and in particular their exponents. An important tool in the discussion is a directed graph that can be associated with each $A \in \pi(K)$, and whose vertices are the extreme rays of K. When K is polyhedral and A is K-primitive, an upper bound for the exponent of A is obtained. This bound generalizes the well known result, due to Wielandt, which asserts that the exponent of an $n \times n$ primitive matrix is at most $n^2 - 2n + 2$.