Orbits of operators tending to infinity

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(Joint work with Vladimír Müller)

Let T be a bounded linear operator. We examine a problem whether there is a unit vector x such that $||T^n x|| \to \infty$. Using Keith Ball's plank theorems, we prove that on a Banach space, a sufficient condition is $\sum_{n=1}^{\infty} \frac{1}{||T^n||} < \infty$, while on a complex Hilbert space, it is $\sum_{n=1}^{\infty} \frac{1}{||T^n||^2} < \infty$. The above results are the best possible. We also show analogous results for weak orbits.